

Study on Reconstruction Algorithm of X-ray Fluorescence Computed Tomography based on L1/2-norm and Expectation-Maximum

Jiang Shanghai*, Yang Shuang, Gu Hong, Shi Shenghui, Luo Binbin, Hu Xinyu, Zou Xue, Zhao Mingfu

Introduction

X-ray fluorescence computed tomography (XFCT) uses the external X-ray to excite the high atomic number (high Z) element inside the sample to radiate fluorescence photon, then detects and records the number of the radiated fluorescence photons of the excited element by the detector and carries out image reconstruction, which is a molecular imaging mode that can be used for medical diagnosis, biomedical research. However, X-ray fluorescence CT (XFCT) has the problems of long scanning time and large radiation dose, which is commonly reduced by reducing the number of projections. But the lack of projection data will lead to the poor quality of reconstructed image. Therefore, for reconstructing high-quality image under sparse projection, this paper introduces L1/2-norm to improve the quality of image reconstruction, which is to achieve the goal of better reconstructed image quality under the same projection condition.

For the regularization problem based on L1/2 norm:

$$\min_x \left[\|P - AX\|_2^2 + \gamma \|X\|_{1/2}^{1/2} \right] \quad (6)$$

Z. Xu et al. proposed a fast method for L1/2 regularization:

$$X = R_{\mu\gamma,1/2} \left(X + \mu A^T (P - AX) \right) \quad (7)$$

where, $R_{\mu\gamma,1/2}$ is operator, namely:

$$R_{\mu\gamma,1/2} = \left(f_{\gamma,1/2}(X_1), f_{\gamma,1/2}(X_2), \dots, f_{\gamma,1/2}(X_N) \right)^T \quad (8)$$

$$f_{\gamma,1/2}(X_i) = \frac{2}{3} X_i \left\{ 1 + \cos \left[\frac{2\pi}{3} - \frac{2}{3} \varphi_\gamma(X_i) \right] \right\} \quad (9)$$

$$\varphi_\gamma(X_i) = \cos^{-1} \left[\frac{\gamma}{8} \left(\frac{|X_i|}{3} \right)^{-\frac{3}{2}} \right] \quad (10)$$

Therefore, the solution of (2) can be obtained by using the fast solution algorithm to solve (4) and the gradient descent algorithm to solve (3).

Numerical simulation

A. Fan-beam XFCT system

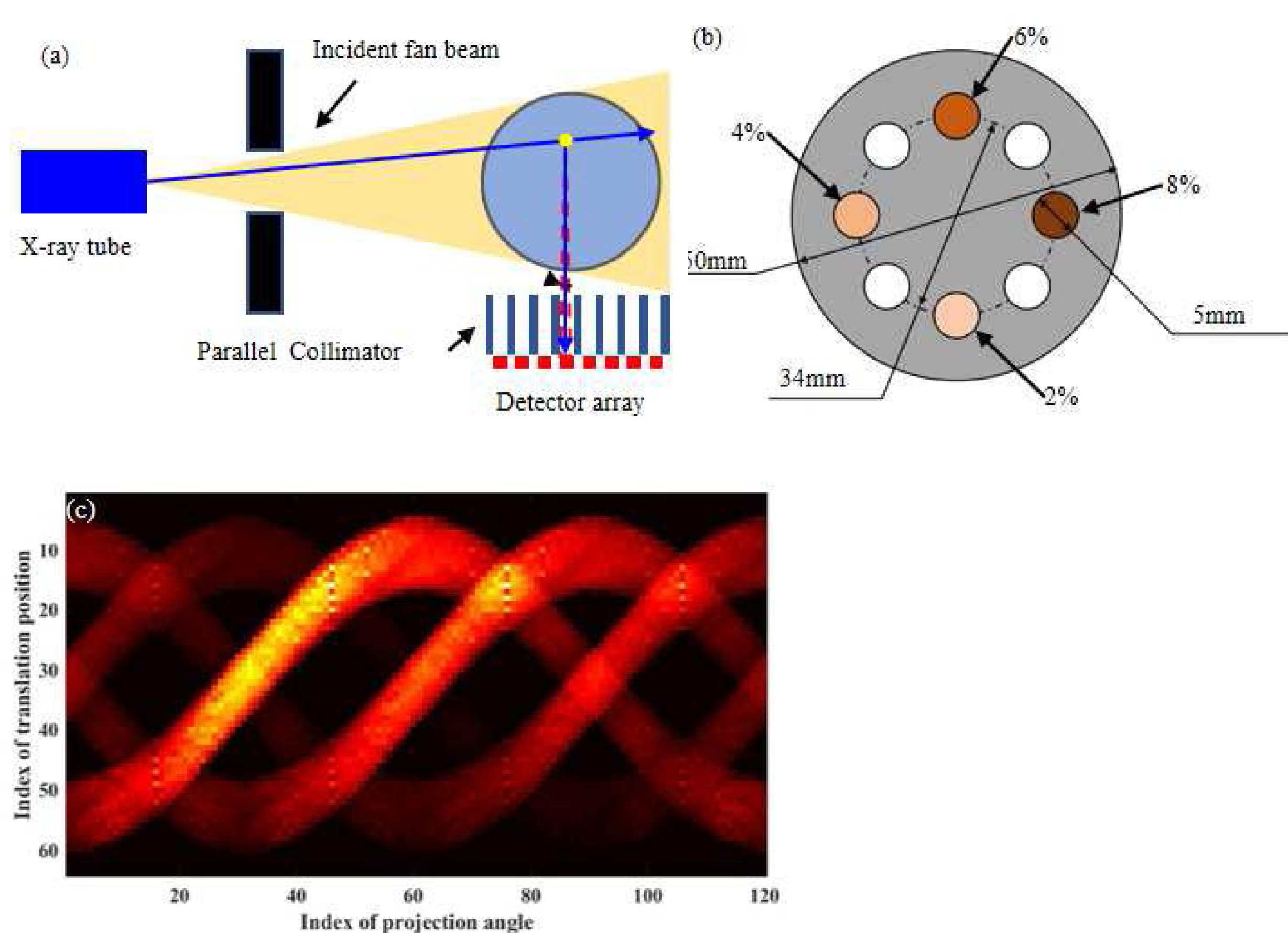


Fig.1 XFCT imaging system. (a) is the schematic diagram of fan beam XFCT imaging system; (b) is the schematic diagram of phantom parameters; (c) is the simulated projection data.

B. CT reconstruction algorithm based on L1/2-norm

The L1/2 norm was introduced to reduce the error and the reconstruction problem can be recast into a regularization problem based on L1/2-norm:

$$X = \arg \min_X \left[\|\nabla X\|_{1/2}^{1/2} + \lambda \|P - AX\|_2^2 \right] \quad (2)$$

Then:

$$X^{k+1} = \arg \min_X \left[\lambda \|P - AX\|_2^2 + \mu \|d^k - \nabla X - P^k\|_2^2 \right] \quad (3)$$

$$d^{k+1} = \min_d \left[\|d\|_{1/2}^{1/2} + \mu \|d - \nabla X^{k+1} - P^k\|_2^2 \right] \quad (4)$$

$$P^{k+1} = P^k + \left(\nabla X^{k+1} - d^{k+1} \right) \quad (5)$$

Reconstruction results and analysis

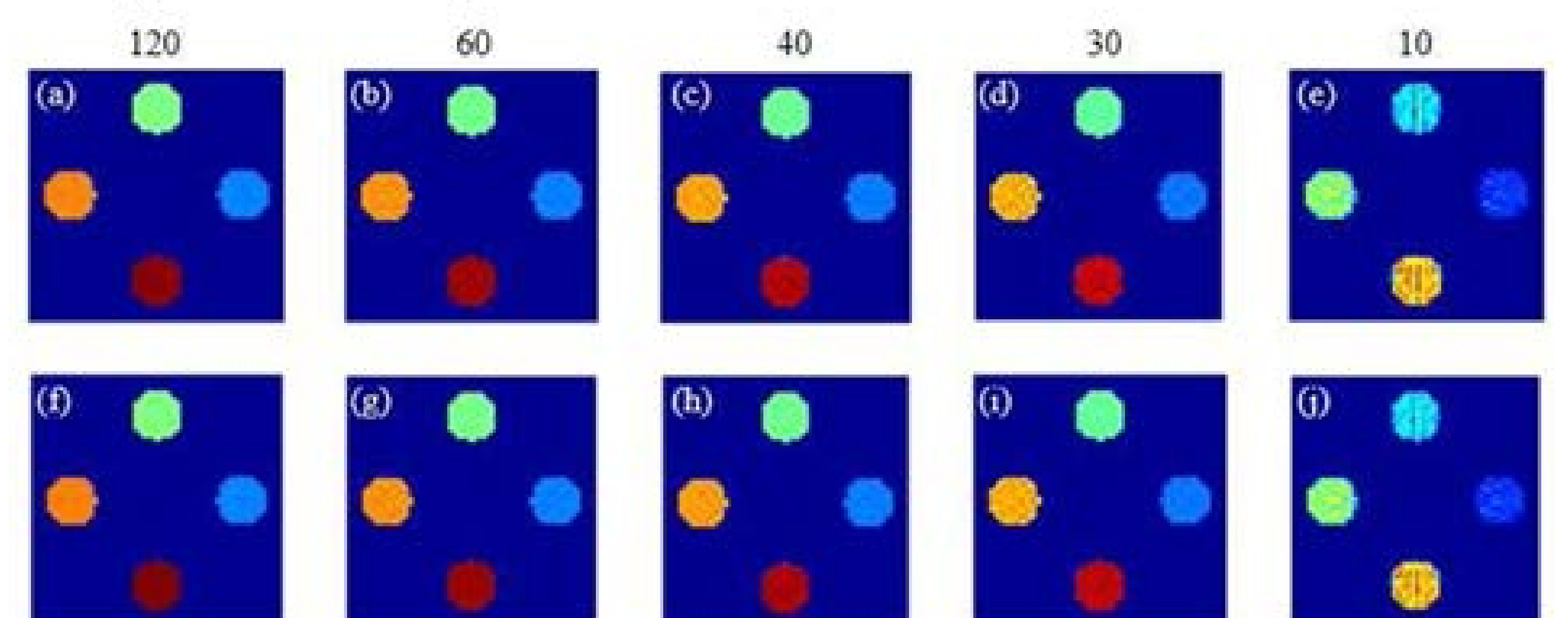


Fig. 2 XFCT images reconstructed by the two algorithms with different projection numbers. (a)-(e) is XFCT image reconstructed by ML-EM algorithm; (f)-(j) is XFCT image reconstructed by MLEM-L1/2 algorithm.

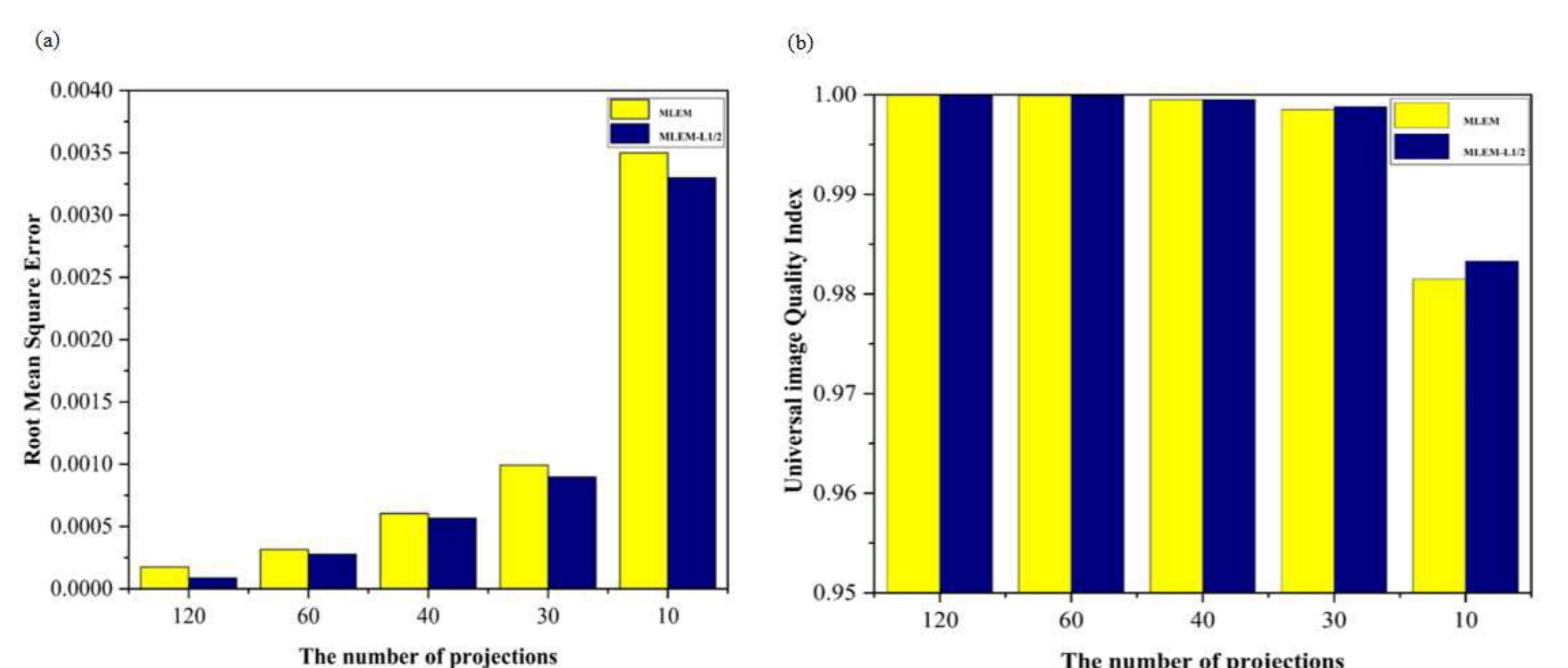


Fig. 3 Evaluation parameters of the two algorithms. (a) RMSE of XFCT image reconstructed by two algorithms under different number of projections; (b) UQI of XFCT image reconstructed by two algorithms under different number of projections

Conclusion

A computed tomography reconstruction algorithm based on L1/2-norm was presented. Numerical simulation indicates that when the number of projections is small and the number of iterations is 100, compared with the traditional MLEM algorithm, the reconstructed image quality of the proposed algorithm is better. Therefore, under sparse projection, MLEM algorithm based on L1/2-norm has more advantages than traditional MLEM algorithm.