Propagation Characteristics of Laguerre-Gaussian Beams with OAM in Atmospheric Turbulence

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ABSTRACT

Based on Kolmogorov turbulence phase aberrations, the OAM measurement probabilities of Laguerre-Gaussian beams in atmospheric turbulence are researched. We derive the functions of OAM measurement probabilities varied with propagation distance of Laguerre-Gaussian beams. In addition, we also find that the Laguerre-Gaussian beam propagating in atmospheric turbulence will be less affected if it has smaller initial OAM topological charge.

INTRODUCTION

The Laguerre-Gaussian (LG) beams with spiral phase factor has a well-defined orbital angular momentum (OAM).Any beam with spiral phase factor carries OAM and OAM does not have to be carried by LG beams. The theoretical achievement is a milestone because it provides a simple way to generate OAM by loading spiral phase on plane waves or Gaussian beams. It also provides a method of measuring OAM by analyzing spiral phase distribution. From then on, the development of OAM was promoted.

Nowadays, one of the important applications of OAM is free-space optical (FSO) communication. It is found that OAM topological charge, as a new dimension, can be used to carry information. The theoretically unlimited values of topological charge, in principle, provide an infinite range of possibly achievable OAM states. And the spatial distribution functions of different OAM states are orthogonal to each other. Therefore, the multiplexing of different OAM states can greatly improve the communication capacity.

Both theoretical and experimental studies show that the Laguerre-Gaussian beams carrying OAM will be disturbed when they propagate in atmospheric turbulence. Atmospheric turbulence can induce scattering and phase aberration, which may lead to energy dispersion and destruction of orthogonality between different OAM states. In this paper, we analyze the propagation characteristics of Laguerre-Gaussian beams and OAM measurement probabilities of Laguerre-Gaussian beams propagating in atmospheric turbulence.

THEORETICAL MODEL

When Laguerre-Gaussian beam modes LG_p^l (l is OAM topological charge, p is radial mode index) pass through the atmospheric turbulence, the complex amplitude of the beams are given by

$$\Psi(r,\theta,z)=U_{lp}(r,\theta,z)\exp\left[i\phi(r,\theta)\right],$$

where the first component is field equation of Laguerre-Gaussian beams, the second component is phase aberrations. Rotational coherence function is defined by

$$C_\phi(r,\Delta\theta)=\left\langle \exp\left\{i\left[\phi(r,\Delta\theta)-\phi(r,0)\right]\right\}\right\rangle,$$

Assuming the refractive index fluctuations to be a Gaussian random process, which allows the standard result

$$\left\langle \exp(ix)\right\rangle=\exp\left(-\frac{1}{2}\left\langle |x|^2\right\rangle\right)$$

to be used, then the rotational coherence function can be written as

$$C_\phi(r,\Delta\theta)=\exp\left[-\frac{1}{2}D_\phi\left(\left|2r\sin\left(\frac{\Delta\theta}{2}\right)\right|\right)\right],$$

For Kolmogorov turbulence phase aberrations, the rotational coherence function can be written as

$$C_\phi(r,\Delta\theta)=\exp\left[-6.88\times 2^{\frac{2}{3}}\left(\frac{r}{r_0}\right)^{\frac{5}{3}}\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right].$$

For Laguerre-Gaussian beams in Kolmogorov turbulence phase aberrations, Fried parameter r_0 can be written as

$$r_0=2.1\times\left[\frac{8}{3\left(a+0.618\Lambda^{11/6}\right)}\right]^{\frac{3}{5}}\left(1.46C_n^2k^2z\right)^{-\frac{3}{5}},$$

where $a=\left(1-\varphi^{8/3}\right)/\left(1-\varphi\right)$, φ is diffractive beam parameter, Λ is Fresnel ratio, C_n^2 is refractive-index structure parameter. Then the rotational coherence function can be written as

$$C_\phi(r,\Delta\theta)=\exp\left[-1.736r^{\frac{5}{3}}aC_n^2k^2z\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right]\exp\left[-1.073r^{\frac{5}{3}}\Lambda^{\frac{11}{6}}C_n^2k^2z\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right]$$

The scattering coefficients of topological charges are the circular harmonic transform of the rotational coherence function

$$\Theta(r,\Delta l)=\frac{1}{2\pi}\int_0^{2\pi}C_\phi(r,\Delta\theta)\exp(-i\Delta l\Delta\theta)\,d\Delta\theta.$$

Substituting rotational coherence function with its specific expression, we can get

$$\Theta(r,\Delta l)=\frac{1}{2\pi}\int_0^{2\pi}\exp\left[-1.736r^{\frac{5}{3}}aC_n^2k^2z\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right]\exp\left[-1.073r^{\frac{5}{3}}\Lambda^{\frac{11}{6}}C_n^2k^2z\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right]\exp(-i\Delta l\Delta\theta)\,d\Delta\theta$$

OAM measurement probabilities in Kolmogorov turbulence phase aberrations are defined by

$$p(l=l_0\pm\Delta l)=\int_0^\infty\left|R(r,z)\right|^2\Theta(r,\Delta l)\,r\,dr.$$

where l_0 is initial topological charge and Δl is interval between l and l_0. $|R(r,z)|^2$ can be substituted with radial intensity profile of Laguerre-Gaussian beams and scattering coefficients can be substituted with its specific expression. Then we can get the relationship between OAM measurement probabilities and propagation distance of Laguerre-Gaussian beams in Kolmogorov turbulence phase aberrations,

$$p(l=l_0\pm\Delta l)=\frac{1}{2\pi}\int_0^\infty\int_0^{2\pi}\left|U_{l_0p}(r,z)\right|^2 r\exp\left[-1.736r^{\frac{5}{3}}aC_n^2k^2z\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right]$$

$$\times\exp\left[-1.073r^{\frac{5}{3}}\Lambda^{\frac{11}{6}}C_n^2k^2z\left|\sin\left(\frac{\Delta\theta}{2}\right)\right|^{\frac{5}{3}}\right]\exp(-i\Delta l\Delta\theta)\,d\Delta\theta\,dr.$$

SIMULATION

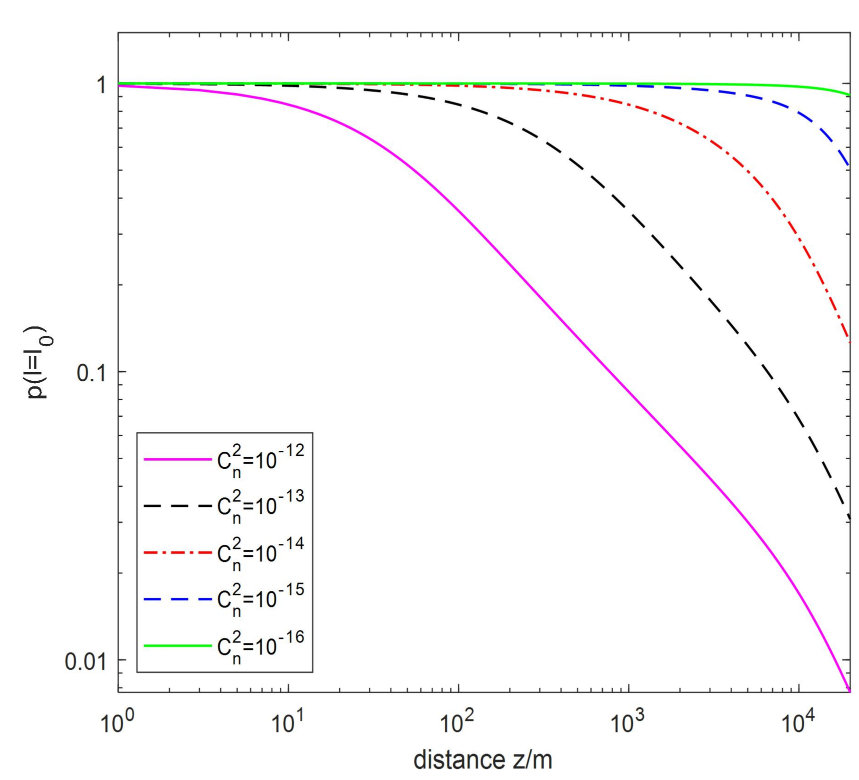

Figure 1. Measurement probabilities of initial topological charge $p(l_0)$.

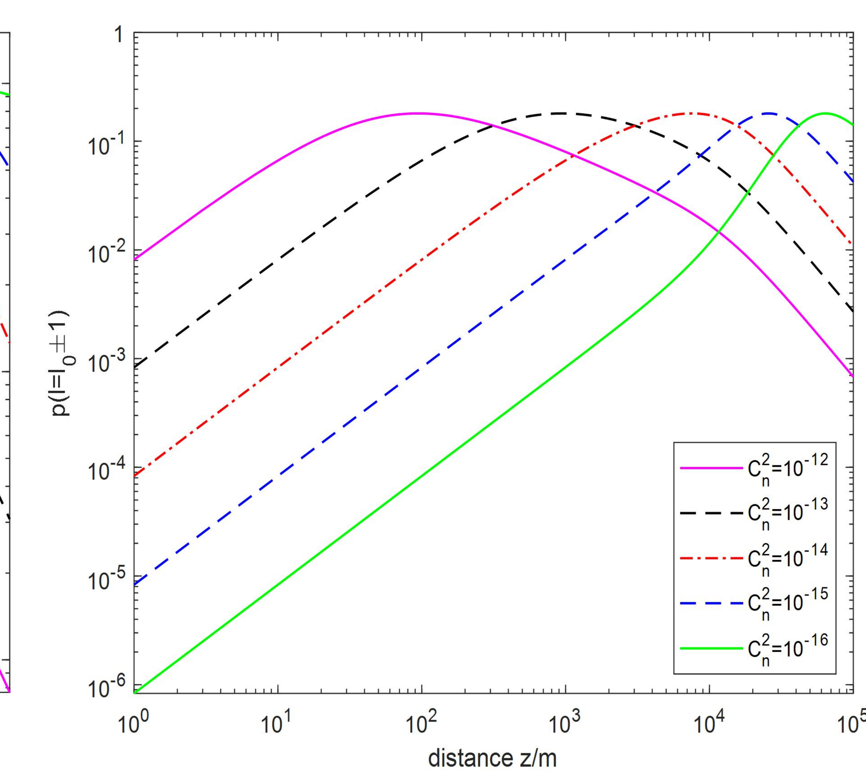

Figure 2. Measurement probabilities of adjacent topological charges $p(l=l_0\pm1)$.

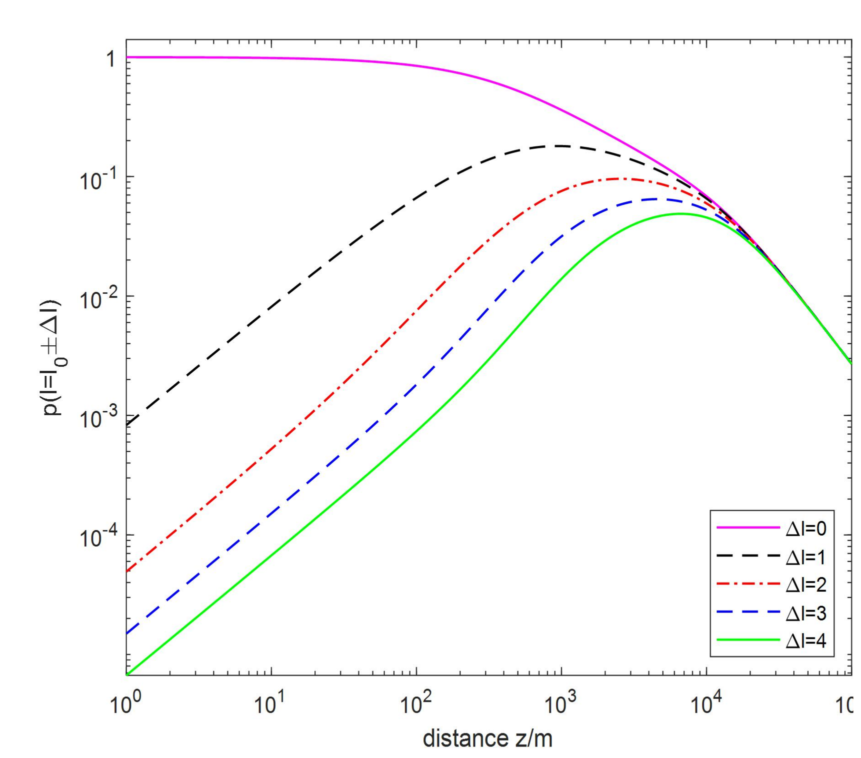

Figure 3. Measurement probabilities of several topological charges $p(l=l_0\pm\Delta l)$.

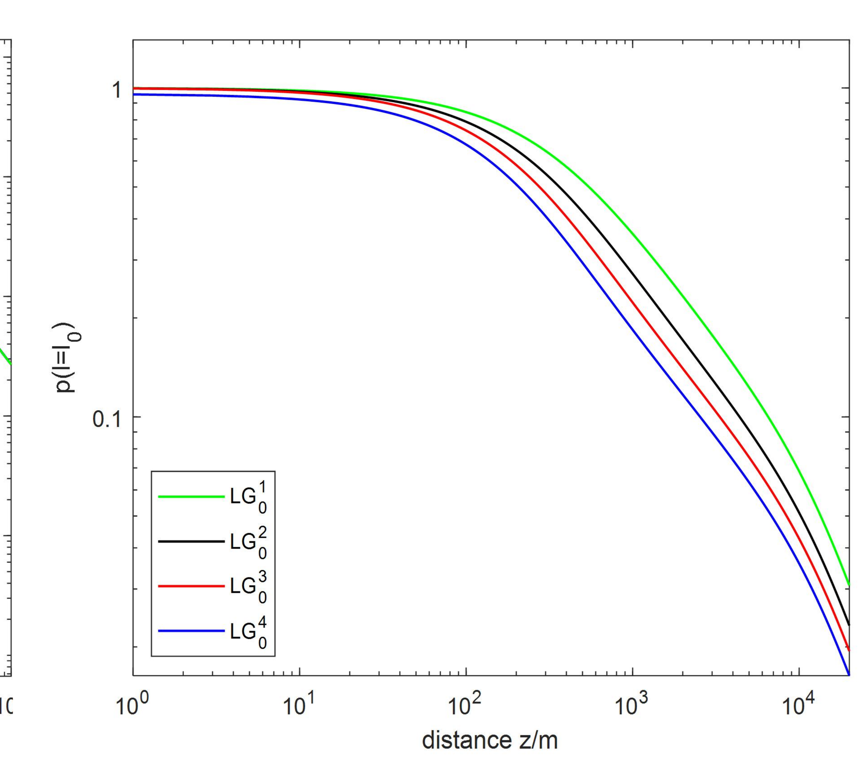

Figure 4. Measurement probabilities $p(l=l_0)$ for different Laguerre-Gaussian beam modes.

CONCLUSION

Based on Kolmogorov turbulence phase aberrations, the OAM measurement probabilities of Laguerre-Gaussian beams in atmospheric turbulence are researched. We derive the functions of OAM measurement probabilities varied with propagation distance of Laguerre-Gaussian beams. For single Laguerre-Gaussian beam mode LG_p^l, with the increase of propagation distance, the measurement probabilities $p(l_0)$ will decrease but $p(l=l_0\pm\Delta l)(\Delta l\neq0)$ will increase first and then decrease. Furthermore, the $p(l=l_0\pm\Delta l)(\Delta l\neq0)$ will reach to the maximum in a longer propagation distance with larger Δl. We conjecture that when the Laguerre-Gaussian beams propagate in atmospheric turbulence, the optical power of the OAM modes with initial topological charges will disperse to the adjacent OAM modes first. Then the decrease at certain propagation distance occurs because the optical power disperses to more and more OAM modes step by step. Besides, we also find that Laguerre-Gaussian beams with smaller initial topological charges will be less affected in atmospheric turbulence.